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by Bernard Gross and John E. Srawley

Lewis Research Center

Cleveland, Ohio

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SUMMARY

A boundary-value-collocation procedure was used in conjunction with the Williams stress function to determine values of the stress-intensity factor K for single edge cracks of various depths in specimens subjected to pure bending. The results are of use in connection with K_{Ic} crack toughness tests, which utilize rectangular-section crack-notch beam specimens loaded in four-point bending, and are in good agreement with published results derived from experimental compliance measurements. The results are expressed in convenient, compact form in terms of the dimensionless quantity $Y^2 = K^2B^2W^3/M^2$, which is a function of relative crack depth a/W only, where B and W are the specimen width and thickness and M is the applied bending moment.

On the assumption that the condition for a valid K_{Ic} test is that the maximum nominal stress at the crack tip should not exceed the yield strength of the material, the K_{Ic} measurement capacity of bend specimens was estimated as a function of a/W. The measurement capacity is proportional to the yield strength and to the square root of the specimen depth, and it is greatest for a/W in the range 0.2 to 0.3.

Values of K for single-edge-notch specimens subjected to combined bending and tension were obtained by superposition of the present results and those of earlier work for specimens loaded in uniform tension. These values are of interest in connection with the use of single-edge-notch specimens that are off-center pin-loaded in tension. It is shown that the K_{Ic} measurement capacity of such specimens is not very sensitive to the eccentricity of loading.

INTRODUCTION

It was shown previously by Gross, Srawley, and Brown (ref. 1) that the value of the stress-intensity factor K for a single edge crack in a flat plate specimen of finite width could be computed accurately by a boundary-

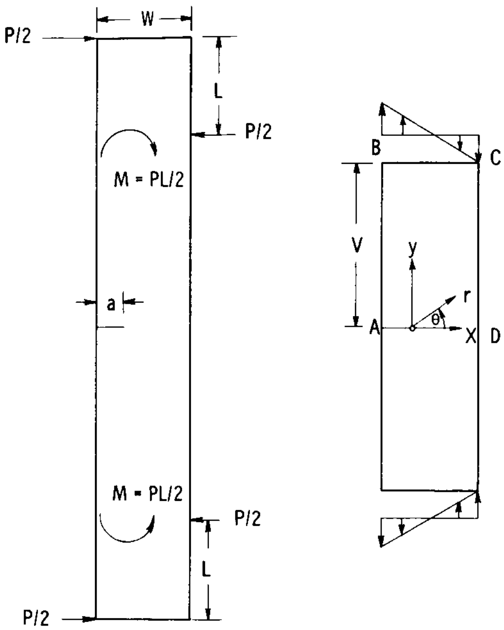

Figure 1. - Single-edge-notch specimen subjected to pure bending.

value-collocation procedure applied to an appropriate stress function. For uniform tensile loading, the computed values of K for various values of the relative crack length a/W were in good agreement with the corresponding values derived from experimental measurements of specimen compliance by Srawley, Jones, and Gross (ref. 2), thus providing confidence in the reliability of the mathematical analysis. The significance of K and its role in the measurement of plane strain crack toughness K_{Ic} are discussed in references 1 and 2, and in greater detail by Srawley and Brown (ref. 3). (All symbols are defined in the appendix).

In the present report, the application of the boundary-value-collocation procedure to the case of single-edge-notch specimens subjected to pure bending is described, and the results are presented. These results are compared with previous results obtained by Bueckner who used a different analytical method (ref. 4), and also with results derived from careful experimental compliance measurements of four-point loaded notched beams by Lubahn (ref. 5). While the results of references 4 and 5 are substantially in agreement, there is sufficient discrepancy between them to warrant a third, independent treatment of the problem in view of the practical importance of the accuracy of K_{Ic} measurements that are conducted with four-point bend specimens.

The use of single-edge-notch specimens loaded in tension through off-center pins has been discussed by Sullivan (ref. 6), and the results of experimental compliance measurements for two positions of the loading pin holes are given in this reference. In the present report, the general case of off-center tension loading is treated by appropriate superposition of the present results for pure bending and the results of reference 1 for uniform tension. This method gives a good approximation to the value of K for any position of the loading pin holes.

In deciding what design of single-edge-notch specimen is to be used for a particular application, an important consideration is the extent to which the K_{Ic} measurement capacity C_{IK} is affected by the design. The K_{Ic} measurement capacity is the largest value of K_{Ic} that could be measured with acceptable accuracy by using a specimen of given width and adequate thickness and a material of given yield strength. For a given specimen design, C_{IK} is proportional to the yield strength and to the square root of the specimen width. In the present report, estimates are obtained of C_{IK} for single-edge-notch specimens loaded in pure bending and in combined bending and tension.

METHOD

The method of analysis consists in finding a stress function χ that satisfies the biharmonic equation $\nabla^4\chi = 0$ and also the boundary conditions at a finite number of stations along the boundary of a single-edge-notched specimen, such as shown in figure 1. The biharmonic equation and the boundary conditions along the crack are satisfied by the Williams stress function (ref. 7). Because of symmetry (fig. 1) the coefficient of the sine terms in the general stress function must be zero, hence

$$\chi(r,\theta) = \sum_{n=1,2\ldots}^{\infty} \left\{(-1)^{n-1} d_{2n-1} r^{n+(1/2)} \left[-\cos\left(n-\frac{3}{2}\right)\theta + \frac{2n-3}{2n+1}\cos\left(n+\frac{1}{2}\right)\theta\right] + (-1)^n d_{2n} r^{n+1}\left[-\cos(n-1)\theta + \cos(n+1)\theta\right]\right\} \quad (1)$$

The stresses in terms of χ obtained by partial differentiation are as follows:

$$\left.\begin{aligned}
\sigma_y &= \frac{\partial^2\chi}{\partial x^2} = \frac{\partial^2\chi}{\partial r^2}\cos^2\theta - 2\frac{\partial^2\chi}{\partial\theta\,\partial r}\frac{\sin\theta\cos\theta}{r} + \frac{\partial\chi}{\partial r}\frac{\sin^2\theta}{r} \\
&\qquad + 2\frac{\partial\chi}{\partial\theta}\frac{\sin\theta\cos\theta}{r^2} + \frac{\partial^2\chi}{\partial\theta^2}\frac{\sin^2\theta}{r^2} \\
\sigma_x &= \frac{\partial^2\chi}{\partial y^2} = \frac{\partial^2\chi}{\partial r^2}\sin^2\theta + 2\frac{\partial^2\chi}{\partial\theta\,\partial r}\frac{\sin\theta\cos\theta}{r} + \frac{\partial\chi}{\partial r}\frac{\cos^2\theta}{r} \\
&\qquad - 2\frac{\partial\chi}{\partial\theta}\frac{\sin\theta\cos\theta}{r^2} + \frac{\partial^2\chi}{\partial\theta^2}\frac{\cos^2\theta}{r^2} \\
-\tau_{xy} &= \frac{\partial^2\chi}{\partial x\,\partial y} = \sin\theta\cos\theta\frac{\partial^2\chi}{\partial r^2} + \frac{\cos 2\theta}{r}\frac{\partial^2\chi}{\partial r\,\partial\theta} - \frac{\sin\theta\cos\theta}{r^2}\frac{\partial^2\chi}{\partial\theta^2} \\
&\qquad - \frac{\sin\theta\cos\theta}{r}\frac{\partial\chi}{\partial r} - \frac{\cos 2\theta}{r^2}\frac{\partial\chi}{\partial\theta}
\end{aligned}\right\} \quad (2)$$

The remaining boundary conditions to be satisfied for the case of pure bending (fig. 1) are as follows:

Along

$$\text{A-B} \qquad \chi = 0; \ \frac{\partial \chi}{\partial x} = 0$$

$$\text{B-C} \qquad \chi = -\frac{12M}{BW^3}\left(\frac{a^3}{6} + \frac{a^2x}{2} + \frac{ax^2}{2} + \frac{x^3}{6}\right) + \frac{6M}{BW^2}\left(\frac{x^2}{2} + ax + \frac{a^2}{2}\right); \ \frac{\partial \chi}{\partial y} = 0$$

$$\text{C-D} \qquad \chi = \frac{M}{B}; \ \frac{\partial \chi}{\partial x} = 0$$

The collocation procedure consists of solving $2m$ simultaneous algebraic equations corresponding to the values of χ and either $\partial\chi/\partial x$ or $\partial\chi/\partial y$ at m selected boundary stations, thus obtaining values for the first $2m$ coefficients in the Williams stress function, the remaining terms being neglected. Only the value of the first coefficient d_1 is needed for the present purpose because this is directly proportional to the stress-intensity factor K. According to Irwin (ref. 8), the stress component σ_y in the immediate vicinity of the crack tip (as r approaches zero) is given by

$$\sigma_y = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

while in terms of the Williams stress function, as r approaches zero

$$\sigma_y = \frac{-d_1}{\sqrt{r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

and similarly for the other stress components, hence,

$$K = -\sqrt{2\pi}\, d_1$$

For given values of M, B, W, a, and V (fig. 1), the value of d_1 computed by the collocation procedure will vary somewhat with m but will approach a limit as m is increased. This is illustrated by the example shown in figure 2 for specimens having a/W equal to 0.3 and various values of V/W. For a given V/W, as the number of boundary stations is increased in steps of three, the computed d_1 oscillates about and converges toward a limit that is close to the average of the five computed values. Accordingly, the value of d_1 that was used to obtain K in each case was the average of five

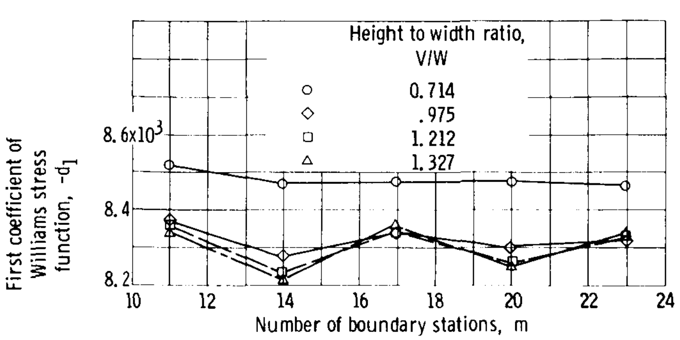

Figure 2. - Value of coefficient d_1 against number of boundary stations. Specimen width, 1 inch; actual crack length, 0.30 inch, applied bending moment, 200 inch-pounds; specimen thickness, 1/16 inch.

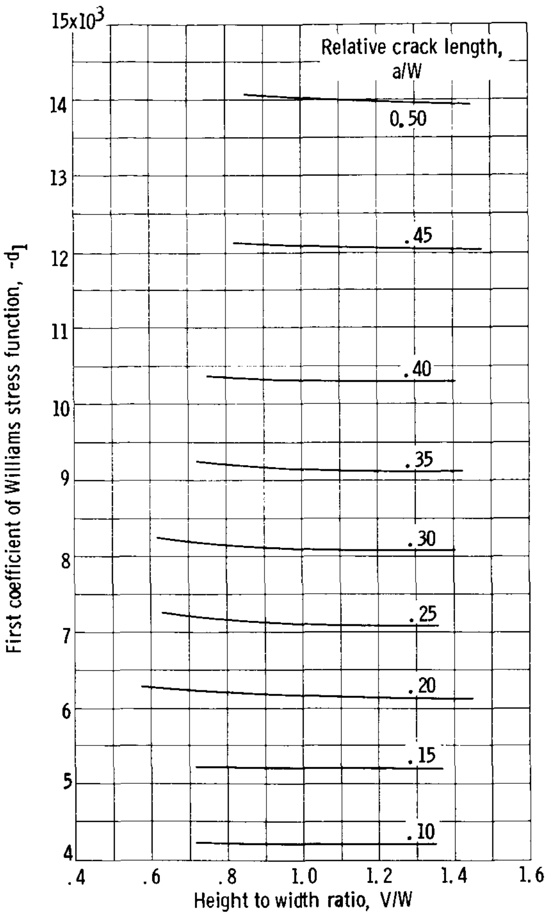

Figure 3. - Single-edge-notch specimen subjected to pure bending. Specimen width, 1 inch; applied bending moment, 200 inch-pounds; specimen thickness, 1/16 inch.

values computed for m equal to 11, 14, 17, 20, and 23. The computations were performed on a digital computer by using double-precision arithmetic (16 significant figures).

A value of K obtained by this method corresponds to a particular set of values of M, B, W, a, and V (fig. 1) selected for convenience of computation. For application, the results are better expressed more generally in terms of the dimensionless quantity $Y^2 = K^2B^2W^3/M^2$ (or in terms of Y), which depends only on the dimensionless ratios a/W and V/W. As will be shown, the effect of V/W on Y^2 is negligible for values of V/W greater than unity, so that for a specimen of adequate length, Y^2 is a function of a/W only. Consequently, a table or graph of Y^2 or Y against a/W is all that is needed for calculation of K for any given values of a, W, B, and M measured in a test.

RESULTS AND DISCUSSION

Pure Bending

The results of the computations for pure bending are summarized in figure 3. Each plotted point represents the average value of d_1 of five values computed for m equal to 11, 14, 17, 20, and 23. The plot shows the dependence of this average value of d_1 on V/W for each of nine values of a/W ranging from 0.1 to 0.5. The relative distance from the crack to the boundary at which a stress distribution corresponding to pure bending was imposed is represented by V/W (fig. 1). In practical terms, it corresponds roughly to the ratio of one-half the minor span to the depth of a four-point loaded beam. It is apparent that the dependence of d_1 on V/W is negligible if V/W exceeds unity. Essentially, the same conclusion was reached for single-edge-notch specimens loaded in uniform tension (ref. 1). Consequently, the K values were calculated from the uniform values of d_1 obtained when V/W was greater than unity. Strictly these K values should be considered to apply to four-point loaded bend specimens only when the minor span is $2W$ or greater.

The final results are given in table I in terms of Y^2 (i.e., $K^2B^2W^3/M^2$) as a function of a/W. The results obtained by a different analytical method (ref. 4) and those derived from experimental compliance measurements (ref. 5) are also tabulated for comparison. Experimental compliance measurements are used to derive values of the strain-energy-release rate with crack extension

TABLE I. - COMPARISON OF RESULTS OF PRESENT WORK WITH

CORRESPONDING RESULTS FROM REFERENCES 4 AND 5

IN TERMS OF DIMENSIONLESS QUANTITY

$K^2B^2W^3/M^2$ AS FUNCTION OF

RELATIVE CRACK LENGTH

Relative crack length, a/W	Results of -		
	Collocation boundary procedure	Reference 4	Reference 5
	$Y^2 = \frac{K^2B^2W^3}{M^2}$		
0.10	12.4	12.2	11.8
.15	18.5	-----	17.4
.20	25.3	25.2	24.2
.25	33.2	-----	32.15
.30	42.8	-----	41.9
.35	55.2	-----	53.9
.40	71.4	-----	68.6
.45	92.7	-----	88.9
.50	123.0	151.2	118.0

$\mathcal{G}$, rather than values of K directly. As discussed previously (ref. 2), the $\mathcal{G}$ values were converted to K values according to the generalized plane stress equation $K^2 = E\mathcal{G}$, where E is Young's modulus.

In reference 4, results are given for only three values of a/W; of these, the two lower results are in excellent agreement with the present results, while the value corresponding to $a/W = 0.5$ is considerably higher than the present result. The agreement between the present results and those of reference 5 is good over the whole range. For the practical purpose of K_{Ic} measurement, the range of a/W between 0.15 and 0.25 is of most importance. In this range there is satisfactory agreement between all three sets of independent results.

The following empirical equation is a compact expression of the present results for values of a/W up to 0.35:

$$Y^2 = \frac{K^2B^2W^3}{M^2}$$

$$= 139 \frac{a}{W} - 221 \left(\frac{a}{W}\right)^2 + 783 \left(\frac{a}{W}\right)^3$$

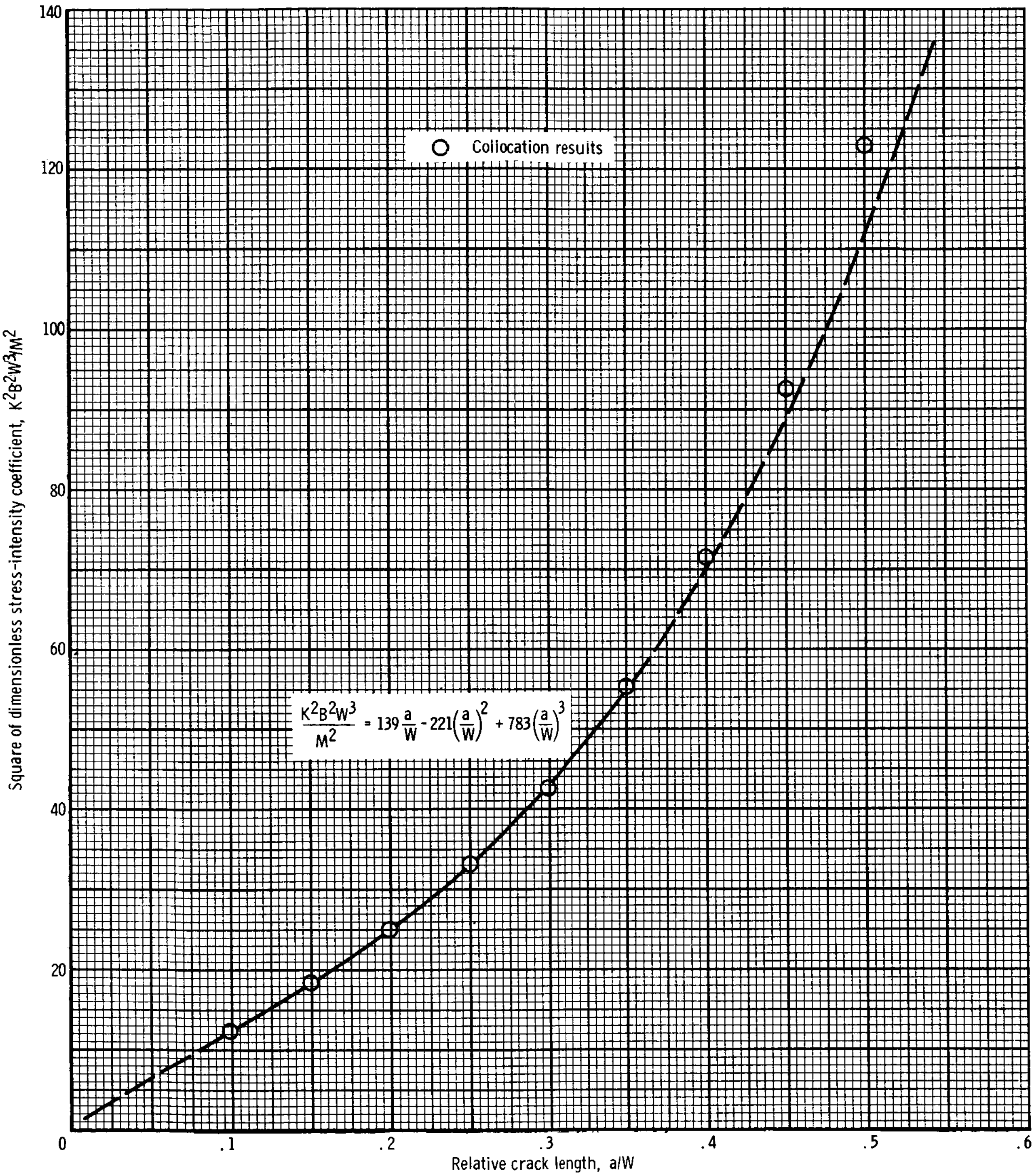

Figure 4. - Curve of least-squares best-fit cubic equation representing results for values of a/W from 0.1 to 0.35. Use of this equation outside this range is not recommended.

The equation was obtained by a least-square best-fit computer program for a cubic in a/W, incorporating the a priori condition that K should be zero when a/W is zero. Only the results for a/W up to 0.35 were used in fitting this equation because it is considered undesirable to use bend specimens having cracks deeper than about 0.35 W. Of the several quantities used to calculate a value of K_{Ic}, the one subject to the greatest uncertainty in measurement is the crack depth. Since the sensitivity of the value of K to a small variation in a/W increases with a/W, it is undesirable to use specimens having large a/W. On the other hand, the efficiency of bend specimens is low when a/W is less than 0.15, as shown in the next section. The optimum range of a/W for bend specimens appears to be between 0.15 and 0.25, and the usual value is 0.2. The curve representing the fitting equation is shown in figure 4, together with the collocation results. The curve is shown dashed in the ranges of a/W from 0 to 0.1 and from 0.35 upwards to emphasize that the equation is not intended to represent the collocation results outside the range of a/W between 0.1 and 0.35.

K_{Ic} Measurement Capacity in Pure Bending

One of the necessary conditions for a meaningful K_{Ic} test is that the specimen width must be sufficient to ensure that the stress field in the vicinity of the crack is sufficiently well represented by that of the assumed linear elastic fracture mechanics model. For reasons that are discussed in reference 3 it will be assumed here that the useful limit of applicability of the model to bend specimens will be reached if the nominal stress at the position of the crack tip reaches the yield strength of the material

$$\frac{6M}{B(W - a)^2} = \sigma_{YS}$$

where σ_{YS} is usually taken to be the 0.2 percent offset tensile yield strength. On this basis, a test result will be valid only if the abrupt crack extension ("pop-in") occurs at a value of M not exceeding $\sigma_{YS}B(W - a)^2/6$. The K value corresponding to this value of M may be defined as the K_{Ic} measurement capacity of a specimen of given dimensions and yield strength, denoted C_{IK}, providing also that the specimen thickness is sufficient as discussed in reference 3. Substituting in the equation

$$Y^2 = \frac{K^2B^2W^3}{M^2}$$

and transposing, we obtain

$$\frac{C_{IK}^2}{\sigma_{YS}^2 W} = \frac{Y^2\left(1 - \frac{a}{W}\right)^4}{36}$$

This equation expresses the K_{Ic} measurement capacity of a bend specimen in terms of the dimensionless quantity C_{IK}^2/σ_{YS}^2W, which is a function of a/W

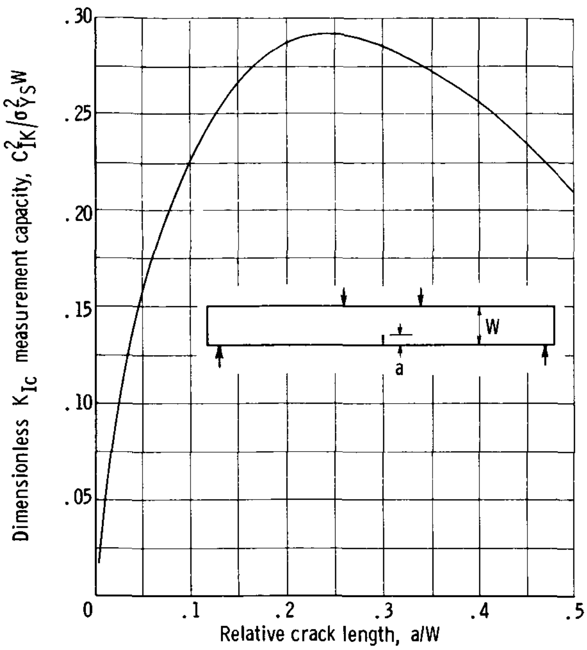

Figure 5. - Dependence of K_{Ic} measurement capacity C_{IK} on relative crack length for specimens in pure bending.

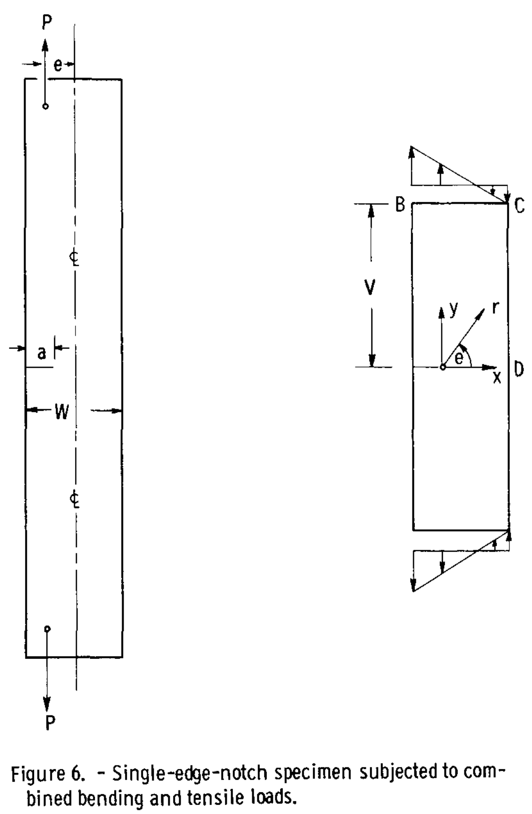

Figure 6. - Single-edge-notch specimen subjected to combined bending and tensile loads.

only since Y is a function of a/W only. For any given value of a/W, there is a unique value of $C_{IK}^2/\sigma_{YS}^2 W$, and hence C_{IK} is proportional to the yield strength of the material and to the square root of the specimen width.

Figure 5 is a plot of $C_{IK}^2/\sigma_{YS}^2 W$ against a/W, which shows that the range of a/W for greatest K_{Ic} measurement capacity is between 0.2 and 0.3. In this range, the C_{IK} for a specimen 3.5 inches deep and of adequate thickness is numerically about equal to the yield strength of the material.

Combined Bending and Tension by Superposition

In the case of a single-edge-notch specimen loaded in tension through pins (fig. 6), the K_{Ic} measurement capacity might be expected to depend on the loading eccentricity parameter e/W as well as on a/W. It is therefore necessary to study this effect of e/W in connection with standardization of design of single-edge-notch tension specimens (ref. 3).

The load P acting through the off-center pins on the single-edge-notch specimen shown in figure 5 is statically equivalent to the combination of an equal load acting along the specimen centerline together with a couple of moment Pe, where e is the distance of the specimen centerline from the line through the pin centers. It should be noted that e is taken to be positive when the pin centers are on the same side of the centerline as the cracked edge and taken to be negative when they are on the other side. By the principle of superposition, the value of each stress component, and therefore the value of K, is equal to the sum of the values

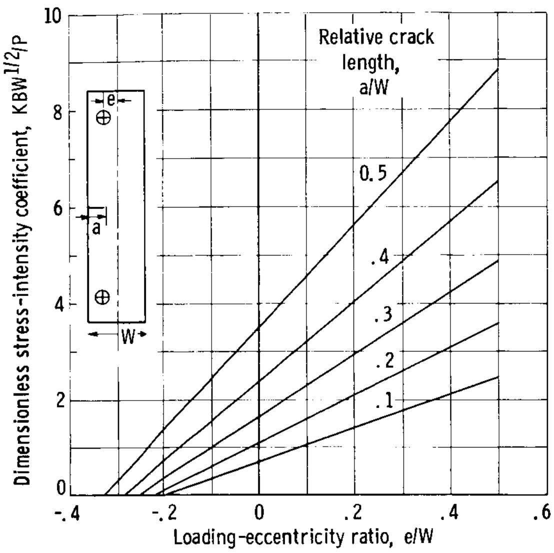

Figure 7. - Dimensionless stress-intensity coefficient as function of loading-eccentricity ratio for single-edge-notch specimens off-center pin-loaded in tension. (Ratio is positive when pin centers are on same side of centerline as cracked edge.)

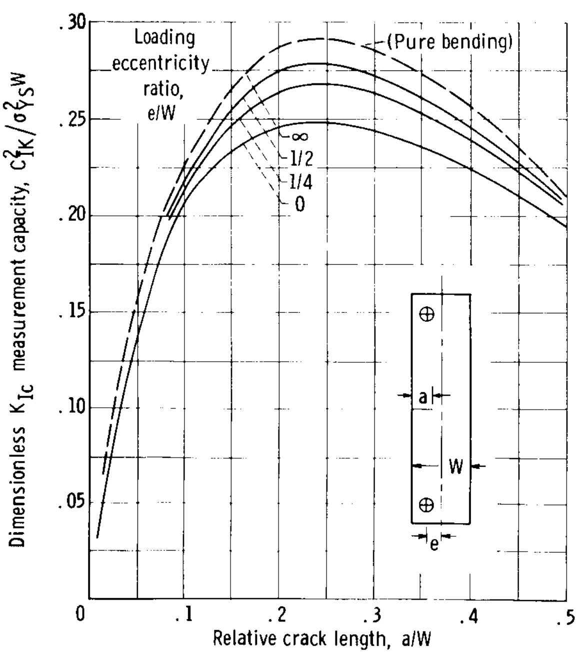

Figure 8. - Dependence of K_{Ic} measurement capacity C_{IK} of single-edge-notch specimens on relative crack length and loading-eccentricity ratio.

that would result individually from the action of P along the centerline and from the action of the couple Pe.

The component of K due to the action of the couple Pe is readily obtained from the present results for pure bending. A good approximation to the component of K due to the centerline tensile load P can be obtained from the results of reference 1. It should be appreciated that the results of reference 1 relate to a specimen loaded uniformly in tension normal to the ends which is not exactly equivalent to a specimen loaded through pins on the centerline. A pin-loaded single-edge-notch specimen bends slightly in proportion to the load, the net effect being that K/P is slightly less for a specimen pin-loaded along the centerline than for the same specimen uniformly loaded at the ends. The magnitude of this effect is discussed in reference 2 in which experimental compliance-measurement results for pin-loaded specimens are compared with the results of reference 1. For the present purpose the magnitude of the effect is negligible.

Figure 7 shows the dimensionless quantity $KBW^{1/2}/P$ against e/W for various values of a/W. By superposition, this quantity is equal to the sum of the component $(KBW^{1/2}/P)_t$, due to the uniform tensile load P, and the component $(KBW^{1/2}/P)_b$ due to the couple Pe. The tensile component was obtained from reference 1 (in that reference the symbol P denotes load per unit thickness, which is P/B in this report). The component $(KBW^{1/2}/P)_b$ is equal to Ye/W, because $M = Pe$, and therefore

$$Y = \left(\frac{K^2B^2W^3}{M^2}\right)^{1/2} = \frac{W}{e}\left(\frac{KBW^{1/2}}{P}\right)_b$$

For a given value of a/W, the values of

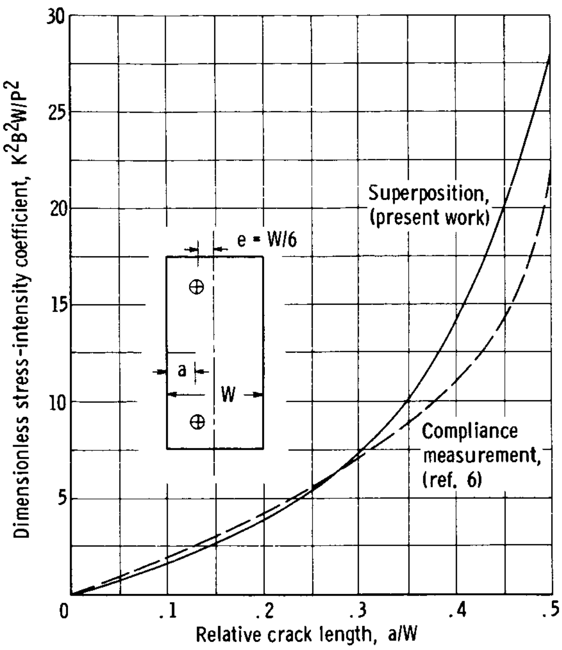

Figure 9. - Comparison of results of present work, for single-edge-notch tension specimens having loading eccentricity ratio of 1/6, with results of experimental measurements.

$(KBW^{1/2}/P)_t$ and of Y are unique, hence,

$$\frac{KBW^{1/2}}{P} = \left(\frac{KBW^{1/2}}{P}\right)_t + Y\frac{e}{W}$$

is a linear function of e/W, as shown in figure 7.

When the same criterion is used for this case as for pure bending, a test result will be valid only if abrupt crack extension occurs at a value of P not exceeding $\sigma_{YS}B(W - a)^2/(W + 2a + 6e)$. This condition was applied in the same manner as previously for pure bending, and values of C_{IK}^2/σ_{YS}^2W were calculated for combined bending and tension on the basis of the values of $KBW^{1/2}/P$ shown in figure 7. Figure 8 shows curves of C_{IK}^2/σ_{YS}^2W against a/W for values of e/W equal to 0, 0.25, and 0.5 and also shows the curve for pure bending from figure 5 for comparison. It is apparent that the measurement capacity is greatest in the range of a/W between 0.2 and 0.3 in all cases. The measurement capacity increases with e/W, but the magnitude of the effect is small, the difference between $C_{IK}/\sigma_{YS}W^{1/2}$ for $e/W = \infty$ (pure bending) and for $e/W = 0$ (uniform tension) being less than 10 percent. Uncertainty about the validity of the basis of comparison is at least of this order, so that for practical purposes it can be assumed that the measurement capacity of single-edge-notch specimens is independent of the manner of loading. For other reasons it is recommended elsewhere (ref. 3) that e/W should be zero for single-edge-notch specimens tested in tension.

A comparison of the results obtained by superposition for $e/W = 1/6$ with the experimental compliance-measurement results of reference 6 for a comparable pin-loaded specimen is shown in figure 9. The results of reference 6 cannot be considered very accurate for reasons discussed in reference 2; nevertheless, the agreement is fairly good for values of a/W up to about 0.3. The increasing discrepancy with increasing a/W beyond 0.3 is, in part, attributable to the fact that the bending, which occurs in the pin-loading of the compliance-measurement specimen, has the effect of slightly reducing the effective bending moment. This effect is neglected in the superposition calculations, as discussed earlier.

CONCLUSIONS

Stress-intensity factors computed by the boundary-collocation procedure for single-edge-notch specimens in pure bending were in good agreement with results derived from experimental compliance measurements. Because of this agreement between two entirely different methods, either result can be used with confidence.

The range of relative crack length a/W within which the K_{Ic} measurement capacity of a bend specimen is greatest between 0.2 and 0.3. This estimate results from the assumption that the nominal stress at the position of the crack tip should not exceed the yield strength in a valid K_{Ic} test.

Stress-intensity factors for single-edge-notched specimens loaded in combined bending and tension can be calculated by appropriate superposition of the available results for uniform tension and for pure bending. The K_{Ic} measurement capacity of single-edge-notch specimens that are loaded off-center in tension is only marginally influenced by the eccentricity of loading.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, October 29, 1964.

APPENDIX - SYMBOLS

a	crack length or depth
B	specimen thickness
C_{IK}	estimate of maximum value of K_{Ic} that can be measured with specimen of given dimensions and yield strength
d_{2n}, d_{2n-1}	coefficients of Williams stress function
E	Young's modulus
e	distance of centerline of single-edge-notched tension specimen from line through loading pin centers
$\mathcal{G}$	strain energy release rate with crack extension per unit length of crack border, or crack extension force
K	stress-intensity factor of elastic stress field in vicinity of crack tip
K_{Ic}	critical value of K at point of instability of crack extension in first or open mode, a measure of plane strain crack toughness of material
L	length of bending moment arm
M	applied bending moment, $PL/2$
m	number of selected boundary stations used in collocation computation
P	total load applied to specimen
r	polar coordinate referred to crack tip
V	distance from crack to boundary at which stress distribution corresponding to pure bending was imposed
W	specimen width
x,y	Cartesian coordinates referred to crack tip
Y	dimensionless stress-intensity coefficient $KBW^{3/2}/M$ that is function of a/W only
θ	polar coordinate referred to crack tip
$\sigma_x, \sigma_y, \tau_{xy}$	stress components

σ_{YS} 0.2 percent offset tensile yield strength

χ stress function

Subscripts:

b bending moment component of Y

t tensile component of Y

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